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Wake-field effect induced by laser multiple pulses

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Using a one-dimensional analysis, the maximum amplitude of the plasma wave induced by a series of square-shaped pulses is analytically calculated for ultraintense laser irradiances. The low-laser amplitude case allows us to recover the beat-wave concept and shows equivalence of the final wake-wave amplitude for one pulse and for a series of pulses with the same power. Large laser amplitudes show the inefficiency of the usual beat-wave process and indicate the need for laser pulses with delays adapted to the wake generation via wake-phase-locked pulses.

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Plasmas can give rise to large-amplitude electric fields with a high phase velocity, via the so-called longitudinal electron plasma wave (EPW). Such a wave is of interest as a particle-accelerator concept since the EPW amplitude can largely exceed the breaking limit of the standard metallic cavity-based accelerator, which is of the order of 30 MV/m. Intense pulsed lasers can be used to amplify a monochromatic EPW. As it propagates, any laser wave packet pushes the electrons forward and backward along the propagation axis (denoted henceforth as the x axis) via the so-called ponderomotive force. This gives the electrons a large longitudinal momentum in the wake of the wave packet, if the laser packet width is the natural oscillation period of the electron fluid, i.e., $2\pi/\omega_{pe}$, where ω_{pe} denotes the plasma radial frequency. As the electric field is bound to the electron charge displacement, large electric fields could be created by using large electron plasma densities n_0. If τ denotes the pulse width in units of picoseconds and n_c is the critical density associated with the incident laser wavelength, the ratio $n_0/n_c=10^{-5}/\tau^2$ corresponds to $\omega_{pe}\tau=2\pi$. Laser technology for exciting EPW's can be divided into two categories: nanosecond pulses with double frequency [1] or picosecond pulses [2]. In the first case, the double-wave beating creates a large number of picosecond-width laser beats, which can induce a resonant mechanism in the plasma; in the second case, only one pulse is supplied, limiting the resonance effect to the width of the pulse. Nowadays, nanosecond pulses give rise, after focusing, to irradiances of a few 10^{16} W/cm^2, whereas multiterawatt subpicosecond laser pulses provide irradiances of a few 10^{18} W/cm^2 [2]. For micrometer wavelengths, the electrons at the laser focus then have a relativistic motion since their quiver momentum, transverse to the laser propagation direction and referred to as $m_e c$, can be written as $a_0=0.85(I_0\lambda_0^2/10^{18})^{1/2}$; m_e is the electron mass, c the light velocity in vacuum, I_0 the laser irradiance in units of W/cm^2, and λ_0 the laser wavelength in μm.

The features of the wake EPW have been studied by using a one-dimensional analysis with the laser group velocity $v_g=c$ and one square-shaped pulse in the so-called quasistatic approximation [3,4]. Recent works, where the assumption $v_g=c$ was relaxed, have shown that the conclusions are correct, provided the plasma is well underdense so that the electron velocities do not reach the wave-breaking limit [5]. Within the previous framework (with $v_g=c$), this paper deals with a sequence of laser pulses and provides both analytical and numerical results concerning the transition between the low-amplitude wave beating and the single large-amplitude laser pulse.

In our model, the plasma is assumed to be cold, homogeneous, and electrically neutral with immobile ions. The laser-plasma interaction is modeled by the conservation equations for the electron density and momentum together with the Poisson equation expressed in the laser pulse frame, by using the normalized coordinate $\zeta=(\omega_{pe}/c)(x-ct)$. This framework allows us to reduce the model to the following second-order nonlinear ordinary differential equation, which describes the EPW scalar potential φ:

$$\frac{d^2\phi}{d\zeta^2}=\frac{1}{2}\left(\frac{\gamma_\perp^2}{(1+\phi)^2}-1\right), \quad (1)$$

where $\phi=e\varphi/m_e c^2$ and $\gamma_\perp$ denotes the time-averaged normalized transverse energy $\sqrt{1+\alpha a^2(\zeta)/2}$; we use $-e$ as the usual electron charge, $a(\zeta)$ as the laser wave amplitude, and $\alpha=1$ and 2 for linear and circular polarization, respectively. If we assume each pulse to be square shaped, i.e., $\gamma_\perp^2(-\zeta_r\leq\zeta\leq\zeta_f)=\gamma_0^2\equiv 1+\alpha a_0^2/2$ (the subscripts f and r stand for the forward and rear fronts, respectively) and $\gamma_\perp^2=1$ outside the pulse, integrating Eq. (1) leads immediately to a relation between ϕ and the normalized electric field $\tilde{E}=eE/m_c c\omega_{pe}=-d\phi/d\zeta$:

$$\left(\frac{d\phi}{d\zeta}\right)^2=C-\left(\frac{\gamma_\perp^2}{1+\phi}+1+\phi\right), \quad (2)$$

where C denotes the constant of integration (CI), to be specifically defined for each pulse and each pulse wake. At front of the first pulse (subscript 1) ${}_1\zeta_f=0$, we have $(\phi,d\phi/d\zeta)=(0,0)$ so that the CI inside the first pulse is ${}_1C_f=\gamma_0^2+1$. A square-shaped pulse is said to be at an optimum, if it maximizes the wake EPW electric field. This occurs when the potential is at a maximum (M) just at the rear of the pulse and, in contrast, the electric field is zero. For the

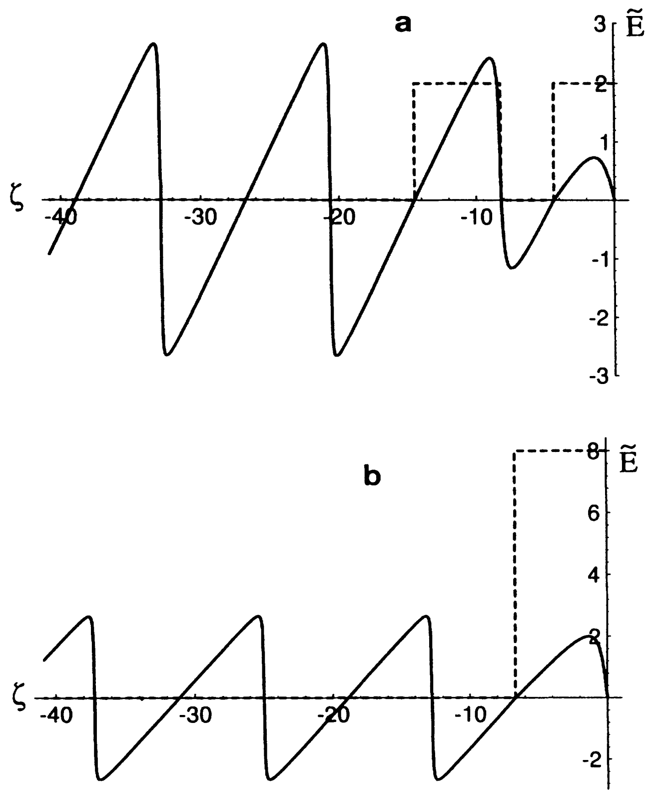

FIG. 1. Pattern of the electric field of the wake wave (solid line) as a function of distance ζ in units of c/ω_{pe}, for (a) two optimized square-shaped pulses with the same amplitude $a_0=2$ and (b) a single pulse with $a_0'=4$. The laser energy profile $a_0^2/2$ $(a_0'^2/2)$ is given by the dotted line.

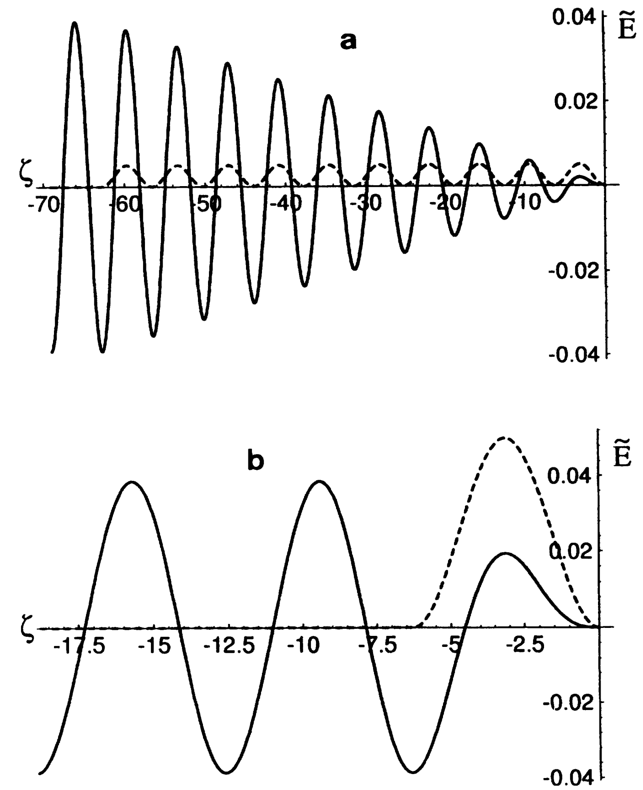

FIG. 2. Pattern of the electric field of the wake wave (solid line) as a function of distance ζ in units of c/ω_{pe}, for (a) 10 $\sin^2$- shaped pulses with $a_0=0.1$ and (b) a single pulse with $a_0'=0.316$. The laser energy profile $a_0^2/2$ $(a_0'^2/2)$ is given by the dotted line. Each pulse width is $2\pi c/\omega_{pe}$.

first pulse, Eq. (2) yields ${}_1\Phi_M=\gamma_0^2-1$. The continuity of the electric field at the rear front lets us infer the CI to be used behind the first pulse, where we use $\gamma_\perp=1$ in Eq. (2), namely, ${}_1C_r=\gamma_0^2+1/\gamma_0^2$. We shall denote ${}_1\zeta_{\rm op}$ the optimum width of the first pulse and ${}_1\lambda_{nl}$ the wavelength of the resulting wake EPW. At the distance ${}_2\zeta=-({}_1\zeta_{\rm op}+{}_1\lambda_{nl}/2)$ the potential is at a minimum ${}_1\phi_m=1/\gamma_0^2-1$ and the electron velocity is at a maximum in the positive x direction. If a second square-shaped pulse has its leading front located at ${}_2\zeta_f={}_2\zeta$, the electrons are pushed again by the front ponderomotive force and their momentum is strongly increased. By using $(\phi,d\phi/d\zeta)=({}_1\phi_m,0)$ as the new initial condition, we find a new CI inside the second pulse: ${}_2C_f=\gamma_0^4+1/\gamma_0^2$. Then, from pulse to wake, we proceed as follows: the CI ${}_1C_f$ given by the leading front of the nth pulse yields the maximum potential ${}_n\phi_M$; at optimum, the latter is used to determine the CI ${}_nC_r$ for the nth wake. We then infer the minimal potential ${}_n\phi_m$. Locating the front of the $(n+1)$th pulse at this potential location, we determine the features of the following pulse-wake couple. To sum up, we successively calculate the following: $[\ ]_{n-1}\rightarrow[{}_nC_f\rightarrow{}_n\phi_M\rightarrow{}_nC_r\rightarrow{}_n\phi_m]\rightarrow[\ ]_{n+1}$. This iteration gives the following CI: ${}_nC_f=\gamma_0^{2n}+1/\gamma_0^{2(n-1)}$ and ${}_nC_r=\gamma_0^{2n}+1/\gamma_0^{2n}$. As a consequence, in the wake of the nth pulse, the minimum and maximum EPW potentials are expressed as

$$ {}_n\phi_m=\frac{1}{\gamma_0^{2n}}-1, \qquad {}_n\phi_M=\gamma_0^{2n}-1. \tag{3}$$

The electric field pattern is found to be symmetric around zero, with the minimum and maximum amplitudes:

$$ {}_n\tilde{E}_M=-{}_n\tilde{E}_m=\gamma_0^n-\frac{1}{\gamma_0^n}. \tag{4}$$

Integrating Eq. (2) leads to an explicit relation between the potential and the position $\zeta(\phi)$. We restrict ourselves here to giving the two main characteristic lengths we can deduce, namely, the optimum width of the nth pulse:

$$ {}_n\zeta_{\rm op}=2\gamma_0^n\mathscr{E}\left(\sqrt{1-(1/\gamma_0^{2(2n-2)})}\right) \tag{5}$$

where $\mathscr{E}$ denotes the complete elliptic integral of the second kind [6,7] and the wavelength of the wake EPW that will follow the nth pulse:

$$ {}_n\lambda_{nl}=4\gamma_0^n\mathscr{E}\left(\sqrt{1-(1/\gamma_0^{4n})}\right). \tag{6}$$

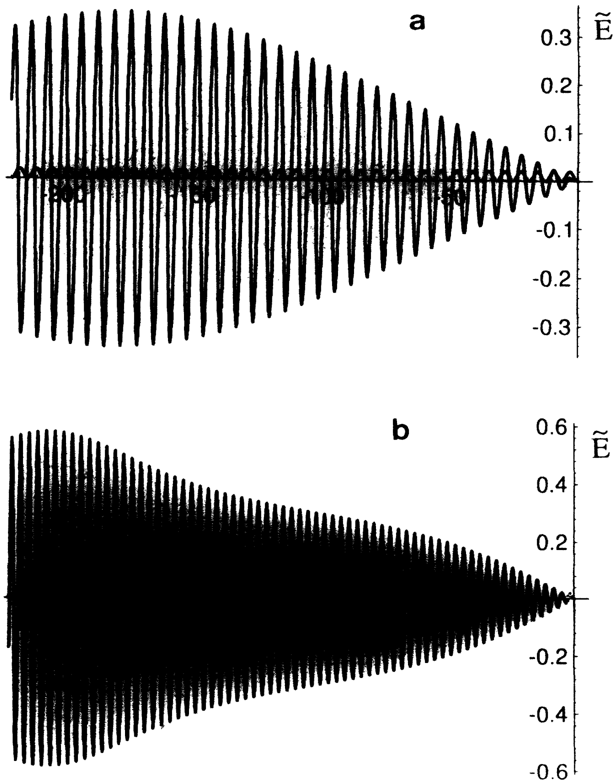

FIG. 3. Pattern of the electric field of the wake wave (solid line) as a function of distance ζ in units of c/ω_{pe} in the case of the beat wave. The individual pulse width is (a) $2\pi c/\omega_{pe}$ and (b) $1.026 \times 2\pi c/\omega_{pe}$. $\sin^2$-shaped pulses are used with $a_0=0.2$. The laser energy profile $a_0^2/2$ is given by the dotted line (close to the x axis).

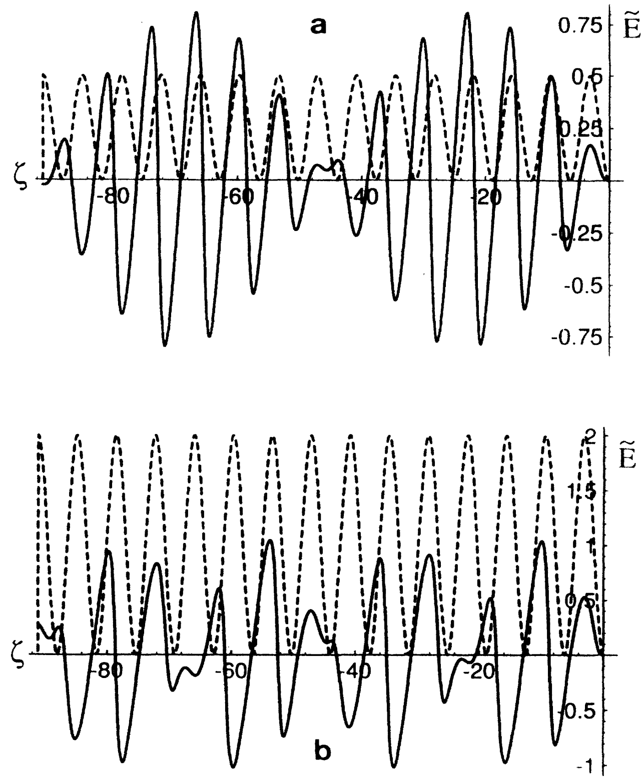

FIG. 4. Pattern of the electric field of the wake wave (solid line) as a function of distance ζ in units of c/ω_{pe}. $\sin^2$-shaped pulses with (a) $a_0=1$ and (b) $a_0=2$ are used with total pulse width $2\pi c/\omega_{pe}$. The laser energy profile $a_0^2/2$ is given by the dotted line.

Comparing Eqs. (5) and (6) shows that for ultraintense irradiances, i.e., $\gamma_0 \gg 1$, the wake wavelength is twice the optimum pulse length, for any pulse of the sequence.

By comparing the EPW electric field induced by a series of n pulses with equal amplitudes a_0 [Eq. (4)] and by a single pulse with strength a_0', i.e., ${}_1\tilde{E}_M=\gamma_0'-1/\gamma_0'$, we find equality for

$$a_0'=\sqrt{2/\alpha}\sqrt{\gamma_0^{2n}-1}, \tag{7}$$

which shows that using multipulses is of interest since their total power is less than the single pulse power. Indeed, for $a_0 \ll 1$ and $a_0 \gg 1$, Eq. (7) leads to $a_0'^2=na_0^2$ and $a_0'^2=(\alpha/2)^{n-1}a_0^{2n}$, respectively. Figure 1 shows the electric field pattern as a function of the distance by numerically integrating Eq. (1) for square-shaped pulses with relativistic irradiances: the cases (a) and (b) correspond to two pulses, optimized as described above, with $a_0=2$ and one single optimized pulse with $a_0'=4$, respectively. Equations (4) and (6) can be seen to be verified.

For unrelativistic irradiances, i.e., $a_0 \ll 1$, Eqs. (4) and (5) yield

$${}_n\tilde{E}_M=-{}_n\tilde{E}_m={}_n\phi_M=-{}_n\phi_m=n\alpha\frac{a_0^2}{2}. \tag{8}$$

The electric field linearly increases as a function of the number of pulses: this characterizes a resonant effect. The electric field of the EPW is the same regardless of whether a series of pulses or one pulse is used, provided the total power is the same. This can be seen in Fig. 2 by comparing the cases (a), i.e., a series of 10 pulses with $a_0=0.1$, and the case (b), i.e., a single pulse with $a_0'=0.316$. Square-shaped pulses with $\zeta_0=\pi$ and the same amplitude a_0 would have given $\tilde{E}_M=0.049$.

A series of laser pulses with individual amplitude a_0 and length ζ_0 can be considered as resulting from the beating of two waves with frequency difference $\Delta\omega/\omega_{pe}=1/\zeta_0$ and amplitudes $a_{1,2}=a_0/2$. When the electron motion becomes relativistic, the plasma frequency is decreased; the resulting phase mismatch increases up to $\pi/2$ (that is a $\lambda_{nl}/4$ distance between the maxima of the pulse profile and the electric field) and kills the resonant effect. Beyond this point the EPW energy is given back to the pulse and the EPW begins to linearly decrease. With $\sin^2$-shaped pulses, Fig. 3(a) shows the EPW saturates with the predicted amplitude [8]:

$$\tilde{E}_{Mb}=(\tfrac{16}{3}\varepsilon)^{1/3}=(\tfrac{4}{3}a_0^2)^{1/3}, \tag{9}$$

with $\varepsilon=a_1a_2/[1+(a_1^2+a_2^2)/2]$ and where the second equality stands for $a_{1,2}=a_0/2\ll 1$. By choosing a pulse width slightly larger than $2\pi/\omega_{pe}$ and so representing a beat wave with a smaller frequency difference $\Delta\omega$, we recover the result predicted in Ref. [8] when $\Delta\omega/\omega_{pe}=1-\frac{1}{2}(9\varepsilon/8)^{2/3}$. In Fig. 3(b), the saturated value is found to be the predicted result:

$$\tilde{E}_M=4(\tfrac{1}{3}\varepsilon)^{1/3}=4(\tfrac{1}{12}a_0^2)^{1/3}. \tag{10}$$

Figure 4 shows the EPW driven by a series of laser pulses as produced by wave beating: the pulses remain regularly spaced with $a_0\geqslant 1$. We can see that wave beating does not induce any gain since the saturation length becomes roughly equal to the pulse width. Figure 4(a) exhibits a smaller value than the predicted one, i.e., $\tilde{E}_{Mb}=1$, due to the increase of the electron mass by transverse oscillation. Figure 4(b) does not show any beating effect: dephasing of the EPW electric field can be clearly seen.

In conclusion, the wake-field effect can be strongly enforced by using a series of laser pulses with only marginally relativistic amplitudes (i.e., irradiances about 10^{18} W/cm^2 at 1 μm wavelength), provided the pulses are delayed and stretched in order to phase-lock them with the electron motion inside the EPW. Experimental tests of the laser-wake-field concept, so far never observed, could thus be made easier.

Note added in proof. We recently became aware of a paper on the wake-field effect by D. Umstadter, E. Esarey, and J. Kim, Phys. Rev. Lett. **72**, 1224 (1994).

One of the authors (G.B.) acknowledges fruitful conversations with E. Esarey, W. Mori, J. M. Rax, and D. Umstadter.

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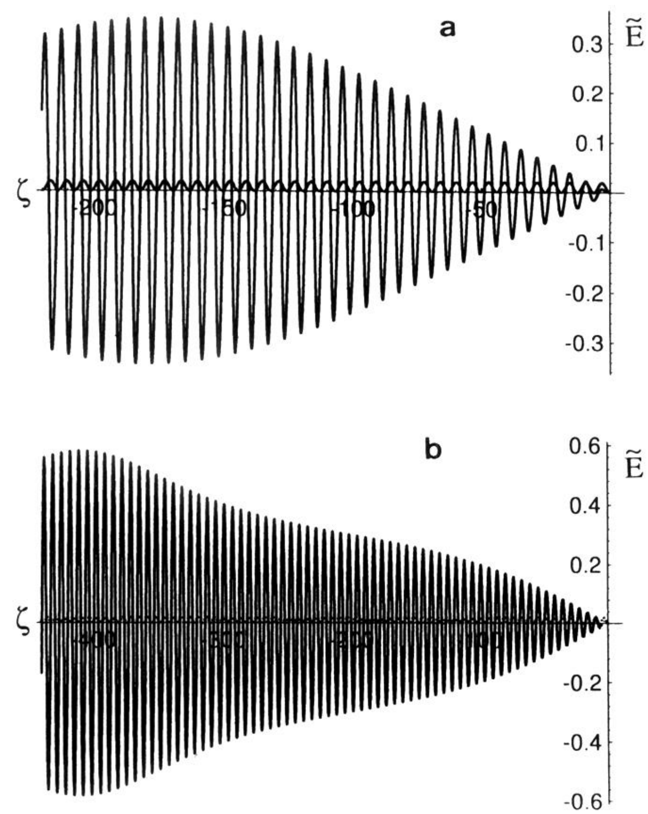

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